

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT2

Branch: Diploma (All)

Semester : 2

Date : 20/04/2019

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a) $\lim_{x \rightarrow 0} \frac{\sin 3x}{2x} = \underline{\hspace{2cm}}$
(A) $3/2$ (B) $2/3$ (C) $1/3$ (D) $1/2$
- b) $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = \underline{\hspace{2cm}}$
(A) ax^{n-1} (B) nx^{n-1} (C) na^{n-1} (D) None of these
- c) $\lim_{x \rightarrow 0} \left(1 + \frac{2}{x}\right)^x = \underline{\hspace{2cm}}$
(A) e^2 (B) e (C) $e^{1/2}$ (D) None of these
- d) If $f(x) = \sin x$ then $f(2\pi) = \underline{\hspace{2cm}}$
(A) 1 (B) 0 (C) -1 (D) 2
- e) $\frac{d(e^{7x})}{dx} = \underline{\hspace{2cm}}$
(A) $7e^x$ (B) e^x (C) $\frac{e^{7x}}{7}$ (D) $7e^{7x}$
- f) $\frac{d(\tan^{-1} x)}{dx} = \underline{\hspace{2cm}}$
(A) $\frac{-1}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{1+x^2}$ (D) $\frac{-1}{1+x^2}$
- g) $\frac{d(\cos x)}{dx} = \underline{\hspace{2cm}}$
(A) $-\sec x$ (B) $\sec x$ (C) $-\sin x$ (D) $\sin x$
- h) $\frac{d(\sin^2 x + \cos^2 x)}{dx} = \underline{\hspace{2cm}}$



(A) 2 (B) 1 (C) 0 (D) None of these

i) $\int \frac{1}{\sqrt{x^2 + 4}} dx = \underline{\hspace{2cm}}$

(A) $\cot^{-1} \frac{x}{2} + c$ (B) $\tan^{-1} \frac{x}{2} + c$ (C) $\log \left| x + \sqrt{x^2 + 4} \right| + c$

(D) none of these

j) $\int e^x dx = \underline{\hspace{2cm}}$

(A) $\log x + c$ (B) $e^x + c$ (C) 1 (D) 0

k) $\int_0^1 \frac{2}{1+x^2} dx = \underline{\hspace{2cm}}$

(A) π (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) None of these

l) Magnitude of $3i + 4j + 5k$ is $\underline{\hspace{2cm}}$.

(A) $2\sqrt{5}$ (B) 0 (C) -6 (D) $5\sqrt{2}$

m) If $\bar{x} = (1, 1, 1)$ and $\bar{y} = (2, -1, 3)$ then $\bar{x} \times \bar{y} = \underline{\hspace{2cm}}$

(A) $(4, -1, 3)$ (B) $(-4, -1, 3)$ (C) $(-4, 1, 3)$ (D) $(4, -1, -3)$

n) If $(1, -2, 3) \cdot (4, 5, k) = 0$ then $k = \underline{\hspace{2cm}}$.

(A) -2 (B) 2 (C) $1/2$ (D) None of these

Attempt any four questions from Q-2 to Q-8

Q-2

Attempt all questions

(14)

a) If $\bar{a} = 3i - j - 4k$, $\bar{b} = -2i + 4j - 3k$ and $\bar{c} = -i + 2j - 5k$

(5)

then find $|\bar{a} + 2\bar{b} - \bar{c}|$.

b) Find unit vector which is perpendicular to $\bar{a} = i + j + k$ and

(5)

$\bar{b} = 2i - 2j + k$.

c) Evaluate: $\lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + \dots + n^3}{n^2(1 + 2 + \dots + n)}$

(4)

Q-3

Attempt all questions

(14)

a) Prove that $\lim_{x \rightarrow \infty} \left[\sqrt{x} (\sqrt{x+p} - \sqrt{x}) \right] = \frac{p}{2}$.

(5)

b) Evaluate: $\lim_{\theta \rightarrow 0} \frac{\operatorname{cosec} \theta - \cot \theta}{\theta}$

(5)

c) Find $\frac{dy}{dx}$ if $y = \frac{\log x}{x}$ at $x = 1$.

(4)

Q-4

Attempt all questions

(14)

a) The equation of motion of a particle is $S = 2t^3 + 3t^2 - 12t + 5$.

(5)

(i) Find velocity at $t = 0$. (ii) Find acceleration at $t = 1$.

b) Find $\frac{dy}{dx}$ if $y = \log \sqrt{\frac{1 + \sin x}{1 - \sin x}}$.

(5)



c) Simplify: $(10i + 2j + 3k) \cdot [(i - 2j + 2k) \times (3i - 2j - 2k)]$. (4)

Q-5

Attempt all questions (14)

a) Prove that angle between two vectors $i + 2j$ and $i + j + 3k$ is (5)

$$\sin^{-1} \left(\sqrt{\frac{46}{55}} \right).$$

b) If $y = e^x \sin x$ then prove that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$. (5)

c) Evaluate: $\lim_{x \rightarrow 2} \frac{x^7 - 128}{x^4 - 16}$ (4)

Q-6

Attempt all questions (14)

a) Constant forces $3i - j + 2k$ and $i + 3j - k$ act on a particle and the particle moves the point $2i + 3j + k$ to the point $5i + 2j + 3k$. Find the work done by the forces. (5)

b) Prove that $\int_0^{\frac{\pi}{2}} \frac{\sec x}{\sec x + \cos ecx} dx = \frac{\pi}{4}$. (5)

c) Find derivative of $y = 3^{4x}$ using first principle. (4)

Q-7

Attempt all questions (14)

a) Evaluate: $\int \left[\sqrt{1 + \sin 2x} + \sqrt{\frac{1 + \cos 2x}{1 - \cos 2x}} \right] dx$ (5)

b) Find the area of the region bounded between curve $y = x^2$ and straight – line $x = 2$. (5)

c) For what value of p , vectors $2i + 3j - k$ and $pi - j + 3k$ are perpendicular to each other? (4)

Q-8

Attempt all questions (14)

a) Evaluate: $\int x^2 \log x dx$ (5)

b) Find $\frac{dy}{dx}$ if $y = x^{\sin^2 x}$. (5)

c) Evaluate: $\int_0^{10} W dx$ Where $W = \frac{3}{4}x \left(1 + \frac{x}{10} \right)$ (4)

